Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2022

Test 1 – (10%)

3.1.7, 3.1.8, 3.1.10 to 3.1.16, 3.2.1 to 3.2.3, 3.2.6, 3.2.7

YI

Time Allowed 20 minutes	**First Name**	**Surname**	**Marks** 26 marks

Circle your Teacher's Name:

Mrs Alvaro	Mrs Bestall	Mrs Fraser-Jones
Mr Gibbon	Mrs Greenaway	Mr Koulianos
Mr Luzuk	Mrs Murray	Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART B – CALCULATOR ALLOWED

QUESTION 5 **(5 marks)**

A particle moves in a straight line so that its position $x(t)$ metres at time t seconds, relative to a fixed position O, is given by $x(t) = t(t-4)^2$. Find the following:

a) The velocity at time t. (1 mark)

$v(t) = 3t^2 - 16t + 16$

✓ Correct derivative

b) The values of t when the particle is instantaneously at rest. (2 marks)

$v(t) = 0$

$t = 4, \frac{4}{3}$

✓ Equates v(t) to zero

✓ Both solutions

c) The acceleration after four seconds. (2 marks)

$a(t) = 2(t-4) + 2(t-4) + 2t$

$= 6t - 16$

$a(4) = 24 - 16$

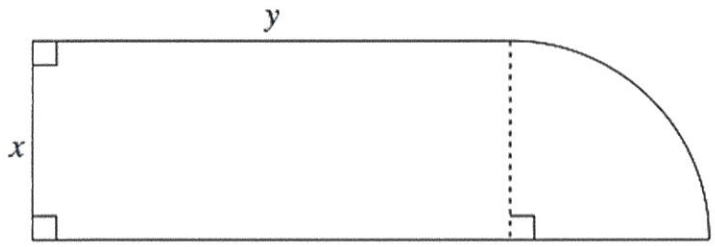
$= 8m/s^2$

✓ Correct a(t), unsimplified is ok

✓ Value with units

QUESTION 6 **(9 marks)**

A landscape gardener wants to build a garden bed in the shape of a rectangle attached to a quarter-circle. Let x and y be the dimensions of the rectangle in metres, as shown in the diagram.



The garden bed is required to have an area of $36m^2$ and to have a perimeter which is as small as possible. Let P metres be the perimeter of the garden bed.

a) Show that $P = 2x + \frac{72}{x}$. (3 marks)

$P = 2x + 2y + \frac{1}{4}(2\pi x)$

✓ States P in terms of x and y

$A = xy + \frac{1}{4}(\pi x^2)$

$36 = xy + \frac{1}{4}(\pi x^2)$

$y = \frac{36}{x} - \frac{\pi x}{4}$

✓ y in terms of x

$P = 2x + 2\left(\frac{36}{x} - \frac{\pi x}{4}\right) + \frac{1}{4}(2\pi x)$

$= 2x + \frac{72}{x} - \frac{2\pi x}{4} + \frac{2\pi x}{4}$

$= 2x + \frac{72}{x}$

✓ Substitutes and simplifies

b) Find the smallest possible perimeter of the garden bed, showing why this is the minimum. State this perimeter and the values of x and y for which it occurs. (6 marks)

$P = 2x + \frac{72}{x}$

$P'(x) = \frac{2x^2 - 72}{x^2}$

✓ Correct first derivative

$P'(x) = 0$

$x = \pm 6$

$\therefore x = 6$

✓ Both values of x

$P''(x) = \frac{144}{x^3}$

$P''(6) = \frac{2}{3}$

$+ve \therefore \min SP$

✓ Correct second derivative

✓ Value of P"(6) and justifies minimum

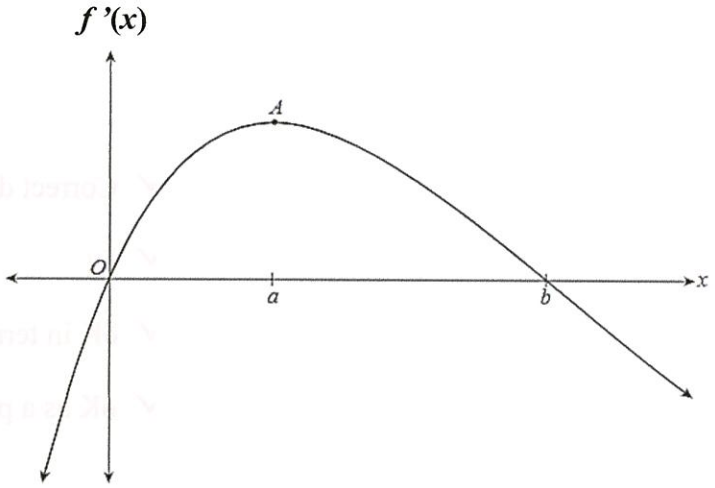
Smallest $P = 24m$ when $x = 6$ and $y = 6 - \frac{3\pi}{2}$.

✓ States smallest P value

✓ States x and y values

QUESTION 7 **(7 marks)**

Consider the function $f(x)$. The graph of its ***derivative***, $y = f'(x)$, is shown below. The graph intersects the x-axis at the origin (O) and at $x = b$. The point A, where $x = a$, is a local maximum of the graph of $y = f'(x)$.



a) Complete the table below by indicating whether $f'(x)$ and $f''(x)$ are positive (+), negative (-) or zero (0) when $x = a$ and when $x = b$. (2 marks)

x	a	b
$f'(x)$	+	0
$f''(x)$	0	-

✓ First row

✓ Second row

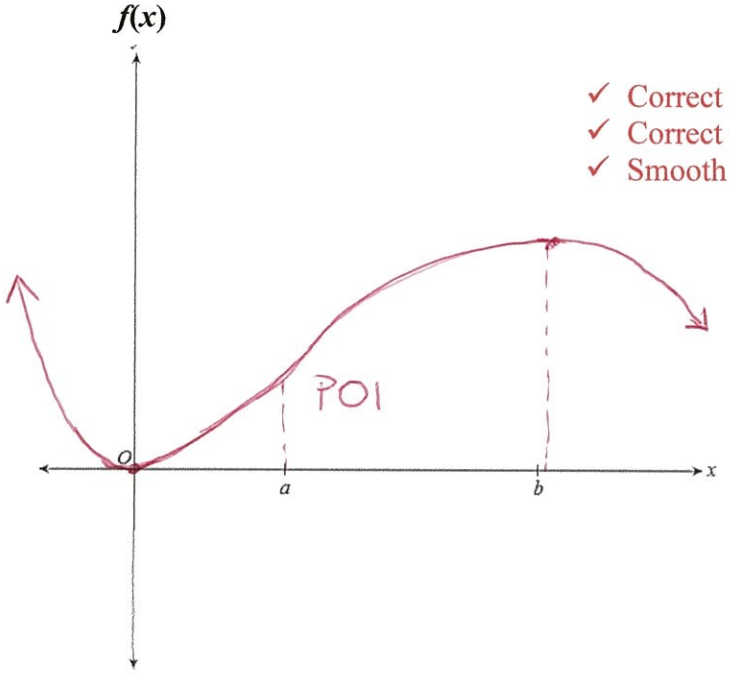
b) What is the nature of Point A? Justify your answer. (2 marks)

A is a point of inflection because there is a change of concavity as A.

✓ States POI

✓ States "change in concavity"

c) On the axes below, sketch a possible graph of $y = f(x)$ that passes through the origin. Clearly show the shape of the graph in the vicinities of the origin, $x = a$ and $x = b$. (3 marks)



✓ Correct min through the origin

✓ Correct max and POI

✓ Smooth curve

QUESTION 8 **(5 marks)**

The kinetic energy K of a body of mass m moving with velocity v is given by $k = \frac{1}{2}mv^2$. If the body's velocity is increased by 1.5%, what is the approximate percentage change in the kinetic energy?

$$K = \frac{mv^2}{2}$$

$$\frac{dK}{dv} = \frac{2mv}{2}$$

✓ Correct derivative

$$\delta v = 0.015v$$

✓ δv

$$\delta K = mv \times 0.015v$$

$$= 0.015mv^2$$

✓ δK in terms of m and v

$$\frac{\delta K}{K} = \frac{0.015mv^2}{\frac{1}{2}mv^2}$$

✓ δK as a proportion of K

$$= 0.03$$

✓ States % increase

There is a 3% increase in the kinetic energy.